

## **SOLUTIONS OF DAILY PRACTICE PROBLEMS**

### **DPP-NO-2A**

1. If  $\log_2 x + \log_4 x = 6$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 2^2} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{2 \log 2} = 6$$

$$\frac{\log x}{\log 2} \left[ 1 + \frac{1}{2} \right] = 6$$

$$\frac{\log x}{\log 2} \times \frac{3}{2} = 6$$

$$\frac{\log x}{\log 2} = 6 \times \frac{2}{3}$$

$$\frac{\log x}{\log 2} = 4$$

$$\log x = 4 \log 2$$

$$\log x = \log 2^4$$

$$x = 2^4$$

$$x = 16$$

2. Given  $\log_x y = 100$  .....(1)

$$\log_2 x = 10$$
 .....(2)

Multiply eq (1) & (2)

$$\log_x y \cdot \log_2 x = 100 \times 10$$

$$\frac{\log y}{\log x} \times \frac{\log x}{\log 2} = 1,000$$

$$\log y = 1,000 \log 2$$

$$\log y = \log 2^{1,000}$$

$$y = 2^{1,000}$$

3. Given

$$(\log_{\sqrt{x}} 2)^2 = \log_x 2$$

$$\left( \frac{\log 2}{\log \sqrt{x}} \right)^2 = \left( \frac{\log 2}{\log x} \right)$$

$$\left( \frac{\log 2}{\log x^{1/2}} \right)^2 = \frac{\log 2}{\log x}$$

$$\left( \frac{\log 2}{\frac{1}{2} \log x} \right)^2 = \frac{\log 2}{\log x}$$

~~$$\left( \frac{2 \log 2}{\log x} \right)^2 = \frac{\log 2}{\log x}$$~~

$$4 \left( \frac{\log 2}{\log x} \right)^2 = \left( \frac{\log 2}{\log x} \right)^1$$

$$4 \frac{\log 2}{\log x} = 1$$

$$4 \log 2 = \log x$$

$$\log 2^4 = \log x$$

$$2^4 = x$$

$$x = 16$$

4.  $\log_4 9 \cdot \log_3 2$

$$\frac{\log 9 \cdot \log 2}{\log 4 \cdot \log 3}$$

$$\frac{\log 3^2 \cdot \log 2}{\log 2^2 \cdot \log 3}$$

$$\frac{2 \log 3 \cdot \log 2}{2 \log 2 \cdot \log 3}$$

$$\frac{2 \log 3 \cdot \log 2}{2 \log 2 \cdot \log 3}$$

$$1$$

5. If  $x = \log_{24} 12$ ,  $y = \log_{36} 24$  and  $z = \log_{48} 36$ , then  $xyz + 1$

$$\log_{24} 12 \times \log_{36} 24 \times \log_{48} 36 + 1$$

$$\frac{\log 12}{\log 24} \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48} + 1$$

$$\frac{\log 12}{\log 48} + 1$$

$$\frac{\log 12 + \log 48}{\log 48}$$

$$\frac{\log 48}{\log(12 \times 48)}$$

$$\frac{\log 48}{\log(576)}$$

$$\frac{\log 48}{\log 24^2}$$

$$\frac{\log 48}{2 \log 24}$$

$$\frac{\log 48}{2 \log 24}$$

$$\frac{\log 48}{2 \log 24}$$

$$\frac{\log 48}{2 \log 24}$$

$$\frac{\log 48}{2 \log 24}$$

$$2 \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48}$$

$$2 \cdot \log_{36} 24 \cdot \log_{48} 36$$

$$2yz$$

6. Given  $\log x = a + b$ ,  $\log y = a - b$

$$\log \left( \frac{10x}{y^2} \right) = \log 10x - \log y^2$$

$$= \log 10 + \log x - 2 \log y$$

$$= 1 + (a + b) - 2(a - b)$$

$$= 1 + a + b - 2a + 2b$$

$$= 1 - a + 3b$$

7. If  $\log 2 = 0.3010$  and  $\log 3 = 0.4771$

$$\text{Then } \log 24 = \log (2 \times 2 \times 2 \times 3)$$

$$= \log 2 + \log 2 + \log 2 + \log 3$$

$$= 3 \log 2 + \log 3$$

$$= 3 \times 0.3010 + 0.4771$$

$$= 0.9030 + 0.4771$$

$$= 1.3801$$

$$\begin{aligned}
8. \quad & \log (1^3 + 2^3 + 3^3 + \dots + n^3) \\
& \log (\sum n^3) \\
& \log \left[ \frac{n(n+1)}{2} \right]^2 \\
& 2 \log \left[ \frac{n(n+1)}{2} \right] \\
& 2 [\log n + \log (n+1) - \log 2] \\
& 2 \log n + 2 \log (n+1) - 2 \log 2
\end{aligned}$$

$$\begin{aligned}
9. \quad & \log_4 9 \cdot \log_3 2 = \frac{\log 9}{\log 4} \cdot \frac{\log 2}{\log 3} \\
& = \frac{\log 3^2}{\log 2^2} \cdot \frac{\log 2}{\log 3} \\
& = \frac{2 \log 3}{2 \log 2} \cdot \frac{\log 2}{\log 3} \\
& = 1
\end{aligned}$$

$$\begin{aligned}
10. \quad & \text{If } \log_3 [\log_4 (\log_2 x)] = 0 \\
& \log_4 (\log_2 x) = 3^0 \\
& \log_4 (\log_2 x) = 1 \\
& \log_2 x = 4^1 \\
& \log_2 x = 4 \\
& x = 2^4 \\
& x = 16
\end{aligned}$$

## DPP-NO-2B

1. Given :  $n = M!$  for  $M \geq 2$

$$\begin{aligned} & \frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \dots + \frac{1}{\log_m n} \text{ or,} \\ & = \log_2 n + \log_3 n + \log_4 n + \dots + \log_m n \\ & = \log_n(2 \times 3 \times 4 \times \dots \times m) \\ & = \log_n(m!) \\ & = \log_n^n \\ & = 1 \end{aligned}$$

2. The integral part of logarithm is called Characteristic and the decimal part of a logarithm is called Mantissa.

3. If  $\log_4(x^2 + x) - \log_4(x + 1) = 2$

$$\log_4 \left\{ \frac{(x^2 + x)}{(x + 1)} \right\} = 2$$

$$\log_4 \left\{ \frac{x(x + 1)}{(x + 1)} \right\} = 2$$

$$\log_4 x = 2$$

$$x = 4^2$$

$$x = 16$$

4.  $\log_5 3 \times \log_3 4 \times \log_2 5$

$$\begin{aligned} & \frac{\log 3}{\log 5} \times \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 2} \\ & \frac{\log 4}{\log 2} \\ & \frac{\log 2}{\log 2} \\ & \frac{2 \log 2}{\log 2} = 2 \end{aligned}$$

5.  $\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}$

$$\log_{60} 3 + \log_{60} 4 + \log_{60} 5$$

$$\log_{60}(3 \times 4 \times 5)$$

$$\log_{60} 60$$

$$1$$

6. If  $\log \left( \frac{x-y}{2} \right) = \frac{1}{2} (\log x + \log y)$

$$2 \log \left( \frac{x-y}{2} \right) = \log x + \log y$$

$$\log \left( \frac{x-y}{2} \right)^2 = \log(xy)$$

$$\left(\frac{x-y}{2}\right)^2 = xy$$

$$\left(\frac{x-y}{4}\right)^2 = xy$$

$$x^2 + y^2 - 2xy = 4xy$$

$$x^2 + y^2 = 4xy + 2xy$$

$$x^2 + y^2 = 6xy$$

7. If  $x = 1 + \log_p qr$ ,  $y = 1 + \log_q rp$ ,  $z = 1 + \log_r pq$

$$x = 1 + \frac{\log qr}{\log p}$$

$$x = \frac{\log p + \log qr}{\log p}$$

$$x = \frac{\log pqr}{\log p}$$

$$\frac{1}{x} = \frac{\log p}{\log pqr}$$

Similarly

$$\frac{1}{y} = \frac{\log q}{\log pqr}$$

$$\frac{1}{z} = \frac{\log r}{\log pqr}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{\log p}{\log pqr} + \frac{\log q}{\log pqr} + \frac{\log r}{\log pqr}$$

$$\frac{\log p + \log q + \log r}{\log pqr}$$

$$\frac{\log pqr}{\log pqr}$$

$$\log pqr$$

$$1$$

8.  $\log x = m + n$  and  $\log y = m - n$

$$\text{Then } \log \left(\frac{10x}{y^2}\right) = \log 10x - \log y^2$$

$$\log 10 + \log x - 2 \log y$$

$$1 + \log x - 2 \log y$$

$$1 + (m + n) - 2(m - n)$$

$$1 + m + n - 2m + 2n$$

$$3n - m + 1$$

9.  $(\log_y x \cdot \log_z y \cdot \log_x z)^3$

$$\left(\frac{\log x}{\log y} \cdot \frac{\log y}{\log z} \cdot \frac{\log z}{\log x}\right)^3$$

$$(1)^3$$

$$1$$

10. If  $x^2 + y^2 = 7xy$

$$x^2 + y^2 + 2xy = 7xy + 2xy$$

$$(x + y)^2 = 9xy$$

Taking log on both side

$$\text{Log}(x + y)^2 = \text{log } 9xy$$

$$2 \text{ log } (x + y) = \text{log } 9 + \text{log } x + \text{log } y$$

$$2 \text{ log } (x + y) = \text{log } 3^2 \text{ log } x + \text{log } y$$

$$2 \text{ log } (x + y) = 2 \text{ log } 3 + \text{log } x + \text{log } y$$

$$2 \text{ log } (x + y) - 2 \text{ log } 3 = \text{log } x + \text{log } y$$

$$2 \left[ \text{log } \frac{(x+y)}{3} \right]$$

$$\text{Log } x + \text{log } y$$

$$\text{Log } \text{log } \frac{(x+y)}{3} = \frac{1}{2} [\text{log } x + \text{log } y]$$

## **DPP-NO-3A**

1. Given :  $\frac{5x-3y}{5y-3x} = \frac{3}{4}$   
 $4(5x - 3y) = 3(5y - 3x)$   
 $20x + 9x = 15y + 12y$   
 $29x = 27y$   
 $\frac{x}{y} = \frac{27}{29}$   
Or,  $x : y = 27 : 29$
  
2. Given : 'α' and 'β' are roots of  $x^2 + x + r = 0$  and  $\alpha^3 + \beta^3 = -6$   
Quadratic equation :  $x^2 + x + r = 0$   
Here  $a = 1$ ,  $b = 1$  and  $c = r$   
Sum of roots :  $\alpha + \beta = \frac{-b}{a} = -1$   
And product of roots :  $\alpha\beta = \frac{c}{a} = r$   
Also,  
 $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$   
on putting the values,  
 $-6 = (-1)^3 - 3r(-1)$   
 $R = -5/3$
  
3. The roots of the equation  
 $Px^2 + qx + r = 0$  are  $\alpha$  &  $\beta$   
Given  $\alpha = r$  then,  
Sum of roots  $\alpha + \beta = \frac{-b}{a}$   
 $r + \beta = \frac{-q}{p} \dots\dots\dots(1)$   
product of roots  
 $\alpha.\beta = \frac{c}{a}$   
 $r.\beta = \frac{r}{p}$   
 $\beta = \frac{1}{p}$
  
4. Let the roots of Q.E  $4x^2 - 6x + P = 0$  is  $\alpha, \beta$   
Here  
 $A = 4$ ,  $b = -6$ ,  $c = p$   
 $\alpha : \beta = 1 : 2$

$$\alpha = k, \beta = 2k$$

$$\alpha + \beta = \frac{-b}{a}$$

$$k + 2k = -\left(\frac{-6}{4}\right)$$

$$3k = \frac{3}{2}$$

$$K = \frac{1}{2} \dots \dots \dots (1)$$

$$\text{And } \alpha \cdot \beta = \frac{c}{a}$$

$$k \cdot 2k = \frac{p}{4}$$

$$2k^2 = \frac{p}{4}$$

$$K = \frac{1}{2}$$

$$2 \cdot \left(\frac{1}{2}\right)^2 = \frac{p}{4}$$

$$P = 2 \times \frac{1}{4} \times 4$$

$$P = 2$$

5. Let  $x^2 - 6x + 10 = y$

$$X^2 - 6x + 10 - y = 0$$

$$X^2 - 6x + (10 - y) = 0$$

$$ax^2 + bx + c = 0$$

we get

$$a = 1, b = -6, c = (10 - y)$$

for real

$$D \geq 0$$

$$b^2 - 4ac \geq 0$$

$$(-6)^2 - 4 \times 1 \times (10 - y) \geq 0$$

$$36 - 40 + 4y \geq 0$$

$$4y \geq 4$$

$$y \geq 1$$

$$y = \{1, 2, 3, \dots, \infty\}$$

minimum value of function = 1

6. If p and q are the roots of the equation  $x^2 - bx + c = 0$

Then

$$p + q = \frac{-b}{a} = \frac{(-b)}{1} = b$$

$$p \cdot q = \frac{c}{a} = \frac{c}{1} = c$$

new roots are  $(pq + p + q)$  &  $(pq - p - q)$



$$\begin{aligned} \text{sum of new roots (s)} &= (pq + p + q) + (pq - p - q) \\ &= pq + p + q + pq - p - q \\ &= 2pq \\ &= 2c \end{aligned}$$

$$\begin{aligned} \text{Product of roots (p)} &= (pq + p + q)(pq - p - q) \\ &= [pq + (p + q)][pq - (p + q)] \\ &= (pq)^2 - (p + q)^2 \\ &= (c^2 - b^2) \end{aligned}$$

Now quadratic equation is

$$X^2 - sx + p = 0$$

$$X^2 - 2cx + c^2 - b^2 = 0$$

7. Say,  $\alpha, \beta$  are the roots of quadratic equation then as per given conditions, we have

$$\text{A.M} = \frac{\alpha + \beta}{2} = 8$$

$$= \alpha + \beta = 16 \text{ (i.e sum of roots)}$$

And given

$$\text{G.M} = \sqrt{\alpha \cdot \beta} = 5$$

$$= \alpha \cdot \beta = 25 \text{ (i.e product of roots)}$$

Required quadratic equation is :-

$$X^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$$

$$X^2 - 16x + 25 = 0$$

8. The equation of lines are

$$4x - 3y = 1 \text{ .....(1)}$$

$$\text{And } 2x - 5y = -3 \text{ .....(2)}$$

Solving these equation (1) & (2) we get

$$X = 1 \text{ and } y = 1$$

Intersection point of lines are (1, 1)

The equation of line parallel to given line

$$4x + 5y = 6 \text{ is given}$$

$$\text{By } 4x + 5y = 8 \text{ .....(1)}$$

It is passing through (1, 1) then from equation (1)

We can say that

The equation of parallel line is

$$4x + 5y = 9$$

$$4x + 5y - 9 = 0$$

9. If  $|x - 2| + |x - 3| = 7$

If  $x - 2 \geq 0$  and  $x - 3 \geq 0$

$$(x - 2) + (x - 3) = 7$$

$$x - 2 + x - 3 = 7$$

$$2x = 7 + 2 + 3$$

$$2x = 12$$

$$x = 6$$

If  $x - 2 < 0$  &  $x - 3 < 0$

$$-(x - 2) - (x - 3) = 7$$

$$-x + 2 - x + 3 = 7$$

$$-2x = 2 \quad x = -1$$

10. Given equation

$$2x^2 + 3x + 7 = 0$$

On comparing with

$$ax^2 + bx + c = 0$$

we get,

$$a = 2, b = 3, c = 7$$

if  $\alpha, \beta$  are the roots of Q.E then

$$\alpha + \beta = \frac{-b}{a} = \frac{-3}{2}$$

$$\alpha \cdot \beta = \frac{c}{a} = \frac{7}{2}$$

$$\alpha\beta^{-1} + \beta\alpha^{-1} = \frac{\alpha^2 + \beta^2}{\alpha\beta} + \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{-3}{2}\right)^2 - 2\left(\frac{7}{2}\right)}{\frac{7}{2}}$$

$$= \frac{-19}{14}$$

## **DPP-NO-3B**

1. Let the present age of grand father = x

The present age of sum of grand sons = y

1<sup>st</sup> condition:

$$X = 3y + 8 \quad \dots\dots\dots(1)$$

After 8 years

The age of grand father = x + 8

The age of sum of grand sons = y + 16

2<sup>nd</sup> condition:

$$(x + 8) = 2 (y + 16) + 10 \quad \dots\dots\dots(2)$$

$$3y + 8 + 8 = 2y + 32 + 10 \quad \text{[from equation (1)]}$$

$$3y + 16 = 2y + 42$$

$$Y = 42 - 16$$

$$Y = 26 \quad \text{in equation (1)}$$

$$X = 3 \times 26 + 8$$

$$= 78 + 8$$

$$= 86$$

Both grand son's are twins so their age =  $\frac{26}{2} = (13)$

The age of grand father when his grand sons was born

$$= 86 - 13$$

$$= 73$$

2.  $X^3 - 7x + 6 = 0$

$$X^3 - x^2 + x^2 - 7x + 6 = 0$$

$$X^2 (x - 1) + x^2 - x - 6x + 6 = 0$$

$$X^2 (x - 1) + x (x - 1) - 6 (x - 1) = 0$$

$$(x - 1) (x^2 + x - 6) = 0$$

$$(x - 1) (x^2 + 3x - 2x - 6) = 0$$

$$(x - 1) [x(x + 3) - 2 (x + 3)] = 0$$

$$(x - 1) (x + 3) (x - 2) = 0$$

If  $x - 1 = 0$  if  $x + 3 = 0$  if  $x - 2 = 0$

$$X = \boxed{X = 1}$$

$$\boxed{X = -3}$$

$$\boxed{X = 2}$$

3. Given quadratic equation

$$4x^2 - 12x + k = 0$$

Comparing from

$$ax^2 + bx + c = 0$$

we get  $a = 4$ ,  $b = -12$ ,  $c = k$

since roots are equal

$$D = 0$$

$$b^2 - 4ac = 0$$

$$b^2 = 4ac$$

$$(-12)^2 = 4 \times 4 \times k$$

$$144 = 16k$$

$$K = 9$$

4. Given lines

$$3x - y = 2 \quad \text{_____ (1)}$$

$$2x + y = 3 \quad \text{_____ (2)}$$

Solving eq. (1) & (2) we get  $x = 1$ ,  $y = 1$

Inter section points of lines is  $(1, 1)$

Since all lines are concurrent so  $(1, 1)$  lies on third lines.

Third line is

$$5x + ay = 3$$

$(1, 1)$  lies on the line

$$5 \times 1 + a \times 1 = 3$$

$$5 + a = 3$$

$$A = -2$$

5. Given lines  $y = 3$  \_\_\_\_\_ (1)

$$X + y = 0 \quad \text{_____ (2)}$$

Solving (1) & (2) we get

$$X = -3, y = 3$$

Inter section points of lines is  $(-3, 3)$

Given lines  $2x - y = 4$

It parallel lines equation

$$2x - y = k \quad \text{_____ (A)}$$

It is passing through  $(-3, 3)$

$$2(-3) - 3 = k$$

$$-6 - 3 = k$$

Putting  $k = -9$  in equation (A) then  $2x - y = -9$

$$2x - y + 9 = 0$$

6. Given  $\alpha + \beta = -2$ , and  $\alpha\beta = -3$

Q.E. is

$$X^2 - (\text{sum of roots})x + \text{product of roots} = 0$$

$$X^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$X^2 - (-2)x + (-3) = 0$$

$$X^2 + 2x - 3 = 0$$

7. Given quadratic equation

$$X^2 + x + 5 = 0$$

If  $\alpha$  &  $\beta$  are the roots of quadratic equation

$$\alpha + \beta = \frac{-b}{a} = \frac{-1}{1} = -1$$

$$\alpha \cdot \beta = \frac{c}{a} = \frac{5}{1} = 5$$

$$\begin{aligned} \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} &= \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta} \\ &= \frac{(-1)^3 - 3 \times 5 \times (-1)}{5} \\ &= \frac{-1 + 15}{5} \\ &= \frac{14}{5} \end{aligned}$$

8.  $\frac{3}{x+y} + \frac{2}{x-y} = -1$  \_\_\_\_\_ (1)

And  $\frac{1}{x+y} - \frac{1}{x-y} = \frac{4}{3}$  \_\_\_\_\_ (2)

Solving (1) & (2), we get (1, 2) by (hits/ trials method)

9. Given cubic equation

$$X^3 + 7X^2 - 21X - 27 = 0$$

By hits/trial -1, 3, -9 are satisfied cubic equation

So, roots of cubic equation are -1, 3, -9.

10. If  $\alpha, \beta$  are the roots of Q.E

$$X^2 - 7x - 9 = 0$$

Comparing from  $ax^2 + bx + c = 0$

We get  $a = 1, b = -7, c = -9$

$$\text{Then } \alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{1} = 7$$

$$\alpha \cdot \beta = \frac{c}{a} = \frac{-9}{1} = -9$$

$$\begin{aligned} \alpha - \beta &= \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} \\ &= \sqrt{(7)^2 - 4 \times (-9)} \\ &= \sqrt{49 + 36} \\ &= \sqrt{85} \end{aligned}$$

### **DPP-NO-3C**

1. Points are (k, 1) (5, 5) and (10, 7)

$$X_1 = k, x_2 = 5, x_3 = 10$$

$$Y_1 = 1, y_2 = 5, y_3 = 7$$

Point are collinear then area of  $\Delta = 0$

$$\text{Area of } \Delta = \frac{1}{2}[x_1 (y_1 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

$$0 = \frac{1}{2}[k(5 - 7) + 5(7 - 1) + 10(1 - 5)]$$

$$0 = [k(-2) + 5(6) + 10(-4)]$$

$$0 = -2k + 30 - 40$$

$$0 = -2k - 10$$

$$-2k = 10$$

$$K = -5$$

2. Let the side of equilateral triangle is x

In  $\Delta ABC$

$$(\text{hypo})^2 = (\text{Base})^2 + (\text{per})^2$$

$$(x - 3)^2 = (x - 4)^2 + (x - 5)^2$$

$$X^2 + 9 - 6x = x^2 + 16 - 8x + x^2 + 25 - 10x$$

$$X^2 - 18x + 41 + 6x - 9 = 0$$

$$X^2 - 12x + 32 = 0$$

$$X^2 - 8x - 4x + 32 = 0$$

$$X(x - 8) - 4(x - 8) = 0$$

$$(x - 8)(x - 4) = 0$$

$$X - 8 = 0 \text{ if } x - 4 = 0$$

X = 8 and x = 4 impossible

Side of the triangle is 8.

3. Given Q.E

$$X^2 + x + 5 = 0$$

$$a = 1, b = 1, c = 5$$

if  $\alpha$  &  $\beta$  are the root of Q.E

$$\alpha + \beta = \frac{-b}{a} = \frac{-1}{1} = -1$$

$$\alpha \cdot \beta = \frac{c}{a} = \frac{5}{1} = 5$$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$\frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}$$

$$\frac{(-1)^3 - 3 \times 5 \times (-1)}{5}$$

$$\frac{-1 + 15}{5}$$

$$\frac{14}{5}$$

4. If  $|A| = 0$  then A is singular Matrix.
5. If A & B are two matrices then  $AB \neq BA$
6. Transpose of a rectangular matrix is a rectangular matrix.

7. If  $\alpha + \beta = -2$  &  $\alpha.\beta = -3$

Q.E is

$$X^2 - (\alpha + \beta)x + \alpha.\beta = 0$$

$$X^2 - (-2)x + (-3) = 0$$

$$X^2 + 2x - 3 = 0$$

8. If  $2^{x+y} = 2^{2x-y} = \sqrt{8}$

$$2^{x+y} = \sqrt{8}$$

$$2^{x+y} = (2^3)^{1/2}$$

$$2^{x+y} = 2^{3/2}$$

On comparing

$$X + y = \frac{3}{2} \quad \text{-----(1)}$$

Add : (1) & (2)

$$X + y = \frac{3}{2} \quad \text{-----(1)}$$

$$2x - y = \frac{3}{2} \quad \text{-----(2)}$$

$$3x = 3$$

$$X = 1$$

Putting  $x = 1$  in equation (1) we get

$$X + y = \frac{3}{2}$$

$$1 + y = \frac{3}{2} \Rightarrow y = \frac{1}{2}$$

$$X = 1, y = \frac{1}{2}$$

9. Given equation

$$X + 2y = 3 \quad \text{-----(1)}$$

$$2x - y = 1 \quad \text{-----(2)}$$

$$Y = 0 \text{ -----(3)}$$

Slope of line (1) is

$$m_1 = \frac{\text{coefficient of } x}{\text{coefficient of } y} = \frac{-1}{2}$$

slope of line (2) is

$$m_2 = \frac{\text{coefficient of } x}{\text{coefficient of } y} = \frac{-2}{-1} = 2$$

$$m_1 \times m_2 = -\frac{1}{2} \times 2$$

$$m_1 \times m_2 = -1$$

both lines are 1 or to each

triangle are also perpendicular

10. Let two numbers are x & y

$$\text{Given } x + y = 13 \text{ -----(1)}$$

$$X^2 + y^2 = 85 \text{ -----(2)}$$

From equation (1)

$$X + y = 13$$

Putting  $y = 13 - x$  in equation (2)

$$X^2 + (13 - x)^2 = 85$$

$$X^2 + 169 + x^2 - 26x = 85$$

$$2x^2 - 26x + 169 - 85 = 0$$

$$2x^2 - 26x + 84 = 0$$

$$2(x^2 - 13x + 42) = 0$$

$$X^2 - 13x + 42 = 0$$

$$X^2 - 7x - 6x + 42 = 0$$

$$X(x - 7) - 6(x - 7) = 0$$

$$(x - 7)(x - 6) = 0$$

$$\text{If } x - 7 = 0 \text{ if } x - 6 = 0$$

$$X = 7 \quad x = 6$$

Putting  $x = 7$  in equation (1) we get  $y = 6$

Putting  $x = 6$  in equation (2) we get  $x = 7$



## DPP-NO-7A

1. Given :- sum of  $n$  – terms of an A.P

$$\text{i.e } S_n = 2n^2 + n$$

$$S_{n-1} = 2(n-1)^2 + (n-1)$$

$$\text{Or } S_{n-1} = 2n^2 - 3n + 1$$

$n^{\text{th}}$  term of this A.P will be

$$T_n = S_n - S_{n-1}$$

$$= (2n^2 + n) - (2n^2 - 3n + 1)$$

$$T_n = 4n - 1$$

$$\text{First term } (T_1) = 4(1) - 1 = 3$$

$$\& \text{ tenth term } (T_{10}) = 4(10) - 1 = 39$$

By question required difference

$$= T_{10} - T_1$$

$$= 39 - 3$$

$$= 36$$

2. Product (P) =  $(243) \times (243)^{1/6} \times (243)^{1/36} \times \dots \dots \dots \infty$

$$= 1 + \frac{1}{6} + \frac{1}{36} + \dots \dots \dots \infty$$

$$= (243)$$

$$= (243)^{\frac{1}{(1-\frac{1}{6})}} \left( \text{in } G P S_{\infty} = \frac{a}{1-r} \right)$$

$$= (243)^{6/5}$$

$$= (3^6)^{6/5}$$

$$= 3^6$$

$$= 729$$

3. Let  $A_1, A_2,$  be the two Arithmetic Means between 68 and 260

68,  $A_1, A_2,$  260 are in A.P

Here,

$$\text{First term } (a) = 68$$

$$\text{Last term } (l) = 260$$

$$\text{Total no. of terms} = 4$$

$$\text{We know, } l = a + (n-1)d$$

$$260 = 68 + (4-1)d$$

$$D = 64$$

$$A_1 = a + d = 68 + 64 = 132$$

4. Given  $P, P^2, P^3, \dots, P^n$

$$\begin{aligned} \text{G.M} &= [P \times P^2 \times P^3 \times \dots \times P^n]^{1/n} \\ &= [P^{(1+2+\dots+n)}]^{1/n} \\ &= \left[ P^{\frac{n(n+1)}{2}} \right]^{1/n} \\ &= P^{\frac{n(n+1)}{2n}} \end{aligned}$$

5. Let  $a$  be the first term of A.P and  $T_n = n^{\text{th}}$  term of A.P and  $d$  be the common ratio of A.P series

$$T_n = a + (n - 1) d$$

$$T_8 = a + (8 - 1) d = 15$$

$$a + 7d = 15 \dots\dots\dots(i)$$

the sum of  $n$  term

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_{15} = \frac{15}{2} [2a + (15 - 1)d]$$

$$= \frac{15}{2} [2a + 14d]$$

$$= \frac{15}{2} \times 2(a + 7d)$$

$$= 15 \times 15 \quad \text{[from eq.(i)]}$$

$$= 225$$

6. Say ' $a$ ' be the first term and ' $d$ ' e the common difference of A.P

$$\text{Given } T_4 = 3T_1 \text{ and } T_7 = 2T_3 + 1$$

$$a + 3d = 3(a)$$

$$\Rightarrow 2a = 3d \dots\dots\dots(1)$$

$$\text{And } T_7 = 2T_3 + 1$$

$$a + 6d = 2(a + 2d) + 1$$

$$a = 2d - 1 \dots\dots\dots(2)$$

solving (1) and (2) we get

$$2(2d - 1) - 3d = 0$$

$$d = 2$$

putting  $d = 2$  in (2) we get

$$a = 2(2) - 1$$

$$a = 3$$

7. In A.P, common difference ( $d$ ) = 2

$$\text{Sum of } n \text{ terms } (S_n) = 49$$

$$7^{\text{th}} \text{ term } T_7 = 13$$

We know that,

$$T_n = a + (n - 1) d$$

$$T_7 = a + (7 - 1) \times 2$$

$$T_7 = a + 6 \times 2$$

$$13 = a + 12$$

$$a = 13 - 12 = 1$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$49 = \frac{n}{2} [2 \times 1 + (n - 1)2]$$

$$49 = \frac{n}{2} \times 2 [1 + n - 1] \quad /$$

$$49 = n^2$$

$$n = \sqrt{49} = 7$$

8. Let first term of G.P is 'a' and common ratio is 'r'

$$T_2 = ar^{2-1} [\because T_n = ar^{n-1}]$$

$$2 = ar \quad (1)$$

$$S_\infty = \frac{a}{1-r}$$

$$8 = \frac{a}{(1-r)}$$

$$a = 8(1-r) \quad (2)$$

putting the value of a in eq (i)

$$2 = 8(1-r).r$$

$$2 = 8r - 8r^2$$

$$8r^2 - 8r + 2 = 0$$

$$4r^2 - 4r + 1 = 0$$

$$4r^2 - 2r - 2r + 1 = 0$$

$$2r(2r - 1) - 1(2r - 1) = 0$$

$$(2r - 1)(2r - 1) = 0$$

$$\text{If } 2r - 1 = 0$$

$$R = \frac{1}{2} \text{ in eq (i)}$$

$$2 = a \times \frac{1}{2}$$

$$a = 4$$

9. Let 1st term of G.P is a and common ratio is r

$$\text{Given } T_6 = 729 \text{ common ratio } (r) = 3$$

$$ar^{6-1} = 729$$

$$ar^5 = 729$$

$$a(3)^5 = 729$$

$$a \times 243 = 729$$

$$a = \frac{729}{243}$$

$$a = 3$$

10. Given

$$I + II + III + \dots \text{ m terms}$$

$$= \frac{1}{9}[9 + 99 + 999 + \dots \text{ m terms}]$$

$$= \frac{1}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \dots \text{ m terms}]$$

$$= \frac{1}{9}[(10 + 100 + 1000 + \dots \text{ m terms}) - (1 + 1 + 1 + \dots \text{ m terms})]$$

$$= \frac{1}{9} \left[ \frac{10(10^m - 1)}{10 - 1} - m \right]$$

$$= \frac{1}{9} \left[ \frac{10}{9} (10^m - 1) - m \right]$$

$$= \frac{1}{81} (10^{m+1} - 10 - 9m)$$

$$= \frac{1}{81} (10^{m+1} - 9m - 10)$$

**DPP-NO-7B**

1.  $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \dots \dots \infty$

$$a = 1, r = \frac{\frac{1}{3}}{1} = \frac{1}{3} < 1$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{1}{1-\frac{1}{3}}$$

$$= \frac{1}{\frac{2}{3}}$$

$$= \frac{3}{2}$$

$$= 1.5$$

2.  $1^3 + 2^3 + 3^3 + \dots \dots \dots + m^3$

$$= \sum m^3$$

$$= \left[ \frac{m(m+1)}{2} \right]^2$$

3. If x, y, z are in G.P

Then x = 1, y = 2, z = 4

$$x^2 + y^2, xy + yz, y^2 + z^2$$

$$= 1^2 + 2^2, 1 \times 2 + 2 \times 4, 2^2 + 4^2$$

= 5, 10, 20 are in G.P

4. If S = n<sup>2</sup>p ans S<sub>m</sub> = m<sup>2</sup>p find S<sub>p</sub> = ?

Now,

$$S = \frac{n}{2}[2a + (n - 1) d]$$

$$n^2 p = \frac{n}{2}[2a + (n - 1) d]$$

$$\frac{2n^2 p}{n} \leq 2a + nd - d$$

$$2a + nd - d = 2np \quad \text{_____ (1)}$$

$$\text{And } S_m = \frac{m}{2}[2a + (m - 1) d]$$

$$m^2 p = \frac{m}{2}[2a + md - d]$$

$$\frac{2m^2 p}{m} \leq 2a + md - d$$

$$2a + md - d = 2mp \quad \text{_____ (2)}$$

Equation (1) – equation (2)

$$2a + nd - d = 2np$$

$$2a + md - d = 2mp$$

- - + -

---

$$nd - md = 2np - 2mp$$

$$d(n - m) = 2p(n - m)$$

$$d = 2p$$

$$d = 2p \text{ in equation (1)}$$

$$2a + m \cdot 2p - 2p = 2mp$$

$$2a = 2mp - 2mp + 2p$$

$$2a = 2p$$

$$a = p$$

the sum of p term of A.P

$$S_p = \frac{p}{2} [2a + (p-1)d]$$

$$= \frac{p}{2} [2 \cdot p + (p-1)2p]$$

$$= \frac{p}{2} [2p + 2p^2 - 2p] \quad /$$

$$= \frac{p}{2} \cdot 2p^2$$

$$= p^3$$

5. We know,

Sum of squares of first, 'n' natural nos. is

$$S_n = \frac{n}{6}(n+1)(2n+1)$$

So for '2n' natural numbers.

Replacing n by 2n in above formula, we get,

$$S_{2n} = \frac{2n}{6}(2n+1)(2(2n)+1)$$

$$= \frac{n}{3}(2n+1)(4n+1)$$

$$\text{Mean of '2n' natural nos.} = \bar{x}_{2n} = \frac{S_{2n}}{2n}$$

$$= \frac{\frac{n}{3}(2n+1)(4n+1)}{2n}$$

$$= 1/6 (2n+1)(4n+1)$$

6. Given ,

$$S_n = 6n^2 + 6n$$

$$S_{n-1} = 6(n-1)^2 + 6(n-1)$$

$$= 6(n^2 + 1 - 2n) + 6n - 6$$

$$= 6n^2 + 6 - 12n + 6n - 6 \quad /$$

$$= 6n^2 - 6n$$

$$T_n = S_n - S_{n-1}$$

$$T_n = (6n^2 + 6n) - (6n^2 - 6n)$$

$$T_n = 6n^2 + 6n - 6n^2 + 6n$$

$$T_n = 12n$$

Fourth term

$$T_4 = 12 \times 4 = 48$$

7. If 'S' be the sum, 'P' the product and 'R' the sum of reciprocals of n terms in G.P then

P is the G.M of  $S^n$  and  $R^n$

$$P^2 = S^n, R^{-n}$$

$$P^2 = \frac{S^n}{R^n}$$

$$P^2 R^n = S^n$$

8. Given

1 + 11 + 111 + ..... to n terms

$$= \frac{1}{9}[9 + 99 + 999 + \dots \text{to n terms}]$$

$$= \frac{1}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \dots \text{to n terms}]$$

$$= \frac{1}{9}[(10 + 100 + 1000 + \dots \text{n terms}) - (1 + 1 + 1 + \dots \text{n terms})]$$

$$= \frac{1}{9} \left[ \frac{10(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{1}{9} \left[ \frac{10^{n+1} - 10}{9} - n \right]$$

$$= \frac{1}{9} \left[ \frac{10^{n+1} - 10 - 9n}{9} \right]$$

$$= \frac{1}{81}(10^{n+1} - 9n - 10)$$

9. Given  $S_n = 3n^2 + 5n$

Putting  $n = 1$ ,  $S_1 = 3(1)^2 + 5(1) = 3 + 5 = 8$

$n = 2$ ,  $S_2 = 3(2)^2 + 5(2) = 12 + 10 = 22$

$S_3 = 3(3)^2 + 5(3) = 27 + 15 = 42$

Then,

$$T_1 = S_1 = 8$$

$$T_2 = S_2 - S_1 = 22 - 8 = 14$$

$$T_3 = S_3 - S_2 = 42 - 22 = 20$$

A.P series is

8, 14, 20, .....

$$a = 8, d = 14 - 8 = 6, T_m = 164$$

$$T_m = a + (m - 1) d$$

$$164 = 8 + (m - 1) \times 6$$

$$164 = 8 + 6m - 6$$

$$6m = 164 + 6 - 8$$

$$6m = 162$$

$$m = \frac{162}{6}$$

$$m = 27$$

10. If a, b, c are in A.P

Then a = 1, b = 2, c = 3

$$= a - b + c$$

$$= 1 - 2 + 3$$

$$= 4 - 2$$

$$= 2$$



## **DPP-NO-7C**

1. Let two number a and b

$$\text{A.M} = \frac{a+b}{2}$$

$$7.5 = \frac{a+b}{2}$$

$$a + b = 15 \text{ _____ (1)}$$

$$\text{G.M} = \sqrt{ab}$$

$$5 = \sqrt{ab}$$

$$25 = ab \text{ _____ (2)} \quad \text{(on squaring both side)}$$

Solving (1) and (2) we get

$$a = 13.09 \text{ and } b = 1.91$$

2. Given series

$$\log x + \log \frac{x^2}{y} + \log \frac{x^3}{y^2} + \dots \dots \dots \text{ n terms}$$

First term (a) =  $\log x$

$$\text{Common difference (d)} = T_2 - T_1$$

$$= \log \left( \frac{x^2}{y} \right) - \log x$$

$$= \log \left( \frac{x^2}{y \cdot x} \right)$$

$$= \log \left( \frac{x}{y} \right)$$

Sum of n terms of A.P

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{n}{2} [2 \cdot \log x + (n - 1) \log \left( \frac{x}{y} \right)]$$

$$= \frac{n}{2} [2 \log x + n \log \left( \frac{x}{y} \right) - \log \left( \frac{x}{y} \right)]$$

$$= \frac{n}{2} [n \log x + 2 \log x - \log \left( \frac{x}{y} \right)]$$

$$= \frac{n}{2} [n \log x + \log x^2 + \log \left( \frac{y}{x} \right)]$$

$$= \frac{n}{2} [n \log \left( \frac{x}{y} \right) + \log \left( x^2 \cdot \frac{y}{x} \right)]$$

$$= \frac{n}{2} [n \log \frac{x}{y} + \log xy]$$

3. If  $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$  are in arithmetic progression then

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{b+c-c-a}{(c+a)(b+c)} = \frac{c+a-a-b}{(a+b)(c+a)}$$

$$\frac{(b-a)}{b+c} = \frac{(c-b)}{(b+a)}$$

$$(b-a)(b+a) = (c-b)(c+b)$$

$$b^2 - a^2 = c^2 - b^2$$

$$b^2 + b^2 = c^2 + a^2$$

$$2b^2 = c^2 + a^2$$

$a^2, b^2, c^2$  are in A.P

4. Given  $a = 5,00,000$ ,  $d = 15,000$ ,  $n = 10$

Total amount after  $n$  years

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{10} &= \frac{10}{2} [2 \times 5,00,000 + (10-1) \times 15,000] \\ &= 5[10,00,000 + 9 \times 15,000] \\ &= 5[10,00,000 + 1,35,000] \\ &= 5[11,35,000] \\ &= 56,75,000 \end{aligned}$$

5. Given series  $50 + 45 + 35 + \dots$   $n$  terms

First term ( $a$ ) = 50

Common difference ( $d$ ) =  $45 - 50 = -5$

Sum of  $n$  terms  $S_n = 0$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$0 = \frac{n}{2} [2 \times 50 + (n-1)(-5)]$$

$$\frac{2}{n} \times 0 = 100 - 5n + 5$$

$$0 = 105 - 5n$$

$$5n = 105$$

$$N = \frac{105}{5} = 21$$

6.  $S_n = 975$ ,  $a = 100$ ,  $d = -5$ ,  $n = ?$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$975 = \frac{n}{2} [2 \times 100 + (n-1)(-5)]$$

$$1,950 = n[200-5n+5]$$

$$1,950 = n[205 - 5n]$$

$$1950 = 205n - 5n^2$$

$$5n^2 - 205n + 1950 = 0$$

$$5(n^2 - 41n + 390) = 0$$

$$n^2 - 41n + 310 = 0$$

$$n^2 - 26n - 15n + 390 = 0$$

$$n(n - 26) - 15(n - 26) = 0$$

$$(n - 26)(n - 15) = 0$$

$$\text{If } n - 15 = 0 \text{ if } n - 26 = 0$$

$$n = 15 \quad n = 26$$

the entire amount will be paid in 15 months

7. If  $a, -3, b, 5, c$  are in A.P

Now  $-3, b, 5$  are in A.P

$$b - (-3) = 5 - b$$

$$b + 3 = 5 - b$$

$$b + b = 2$$

$$2b = 2$$

$$b = 1$$

and  $b, 5, c$  are in A.P

$$5 - b = c - 5$$

$$5 - 1 = c - 5$$

$$4 = c - 5$$

$$c = 4 + 5$$

$$c = 9$$

8. Number b/w 10 and 1000 which are divisible by 11 are 110, 121, 132, -----, 990

Here,  $a = 110, d = 121 - 110 = 11, l = 990$

$$\text{Then, } n = \frac{l - a + d}{d} = \frac{990 - 110 + 11}{11} = \frac{891}{11} = 81$$

Sum of  $n$  term of A.P

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_n = \frac{81}{2} [2 \times 110 + (81 - 1)11]$$

$$= \frac{81}{2} [220 + 80 \times 11]$$

$$= \frac{81}{2} [220 + 880] = 44550$$

9. Given,

$$S_n = (3n^2 - n)$$

$$n = 1$$

$$S_1 = 3(1)^2 - 1 = 3 - 1 = 2$$

$$n = 2, S_2 = 3(2)^2 - 1 = 12 - 1 = 11$$

$$n = 3, S_3 = 3(3)^2 - 1 = 27 - 1 = 26$$

$$T_1 = S_1 = 2$$

$$T_2 = S_2 - S_1 = 11 - 2 = 9$$

$$T_3 = S_3 - S_2 = 26 - 11 = 15$$

$$\text{First term of series} = T_1 = 2$$

10. Let first term and common difference of A.P is  $a$  and  $d$

$$\text{Given } T_4 + T_{12} = 8$$

$$a + 3d + a + 11d = 8$$

$$2a + 14d = 8$$

$$S_{15} = \frac{15}{2} [2a + (15 - 1)d] \quad \{S_n = \frac{n}{2} [2a + (n - 1)d]\}$$

$$S_{15} = \frac{15}{2} [2a + 14d] = \frac{15}{2} \times 8 = 60$$

### **DPP-NO-8A**

- No. of different word can be formed from the letters of 'LIBERTY'  
 $= 7!$   
 $= 5040$
- No. of ways can a family consist of three children have different birthday in a leap years =  
 $366 \times 365 \times 364$

3. If  ${}^{1000}C_{98} = {}^{999}C_{97} + {}^XC_{901}$   
 By properties  ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$   
 ${}^{999}C_{97} + {}^XC_{901} = {}^{1000}C_{98}$  .....(1)  
 ${}^{999}C_{97} + {}^{999}C_{98} = {}^{999+1}C_{98}$   
 ${}^{999}C_{97} + {}^{999}C_{98} = {}^{1000}C_{98}$   
 ${}^{999}C_{97} + {}^{999}C_{999-98} = {}^{1000}C_{98}$   
 ${}^{999}C_{97} + {}^{999}C_{901} = {}^{1000}C_{98}$  .....(2)

Comparing (1) & (2) we get

$X = 999$

- The no. of permutation of 'n' item taken 3 at a time =  ${}^nC_3$   
 The no. of permutation of 'n -1' item taken at a time =  ${}^{n-1}C_3$

Given

$6 \cdot {}^nC_3 = 7 \cdot {}^{n-1}C_3$   
 $6 \cdot \frac{|n}{|n-3} = 7 \cdot \frac{|n-1}{|n-1-3}$   
 $6 \times \frac{n|n-1}{n-3|n-4} = 7 \cdot \frac{|n-1}{|n-4} \Rightarrow 6n = 7n - 21$

- If  ${}^6P_r = 360$

$\frac{6!}{(6-r)!} = \frac{360}{1}$

$360 \cdot (6 - r)! = 6!$

$360 \cdot (6 - r)! = 720$

$(6 - r)! = \frac{720}{360}$

$(6 - r)! = 2$

$(6 - r)! = 2!$

On comparing  $6 - r = 2$

$r = 4$

6. No. of English Book = 5

No. of Tamil Book = 4

No. of Hindi Book = 3

|   |  |  |
|---|--|--|
| E <sub>1</sub> E <sub>2</sub> E <sub>3</sub> E <sub>4</sub><br>E <sub>5</sub> | T <sub>1</sub> T <sub>2</sub><br>T <sub>3</sub> T <sub>4</sub> | H <sub>1</sub><br>H <sub>2</sub><br>H <sub>3</sub> |
| 1   | 2  | 3  |

No. of ways =  $5! \times 4! \times 3! \times 3! = 1,03,680$

7. Total men = 5, total women = 4

OE OE OE OE O

1 2 3 4

No. of ways =  ${}^4P_4 \times 5!$

=  $4! \times 5!$

=  $24 \times 120$

= 2,880

8.

|                   |     | Th |   | H |   | T |   | U |       |
|-------------------|-----|----|---|---|---|---|---|---|-------|
| 1                 | (3) | 1  | × | 6 | × | 5 | × | 4 | = 120 |
| 2                 | (5) | 1  | × | 6 | × | 5 | × | 4 | = 120 |
| 3                 | (7) | 1  | × | 6 | × | 5 | × | 4 | = 120 |
| 5                 | (8) | 1  | × | 6 | × | 5 | × | 4 | = 120 |
| 7                 | (9) | 1  | × | 6 | × | 5 | × | 4 | = 120 |
| 8                 |     |    |   |   |   |   |   |   |       |
| 9                 |     |    |   |   |   |   |   |   |       |
| Total No. of ways |     |    |   |   |   |   |   |   | = 600 |

9. Total Friend = 10

No. of Relative = 6

No. of friend = 4

No. of ways to invite five guest such that three of them are his relatives

=  ${}^6C_3 \times {}^4C_2$

=  $\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1}$

=  $20 \times 6$

= 120

10. Total question = 10

No. of Mathematics questions = 6

No. of Statistics questions = 4

No. of ways at least one question of mathematics =  $(2^6 - 1) = (64 - 1) = 63$

No. of ways at least one question of statistics =  $(2^4 - 1) = (16 - 1) = 15$

Total no. of ways =  $63 \times 15 = 945$

## **DPP-NO-8B**

1. No. of arrangement when one particular thing is always taken is given by  $r \cdot {}^{n-1}P_{r-1}$

$$\text{Here, } n = 12 \quad r = 5$$

$$5 \cdot {}^{12-1}P_{5-1} = 39,600$$

2. No. of ways =  ${}^n P_r$

$$\text{Here } n = 5, r = 3$$

$$= {}^5 P_3$$

$$= \frac{|5}{|5-3}$$

$$= \frac{5 \times 4 \times 3 |2}{|2}$$

$$= 60$$

3. No. of ways =  $n^r = 2^{12} = 4,096$

4. In word "VIOLENT"

$$\text{No. of vowel} = 3 \text{ (I, O, E)}$$

$$\text{No. of consonant} = 4 \text{ (V, L, N, T)}$$

$$\text{Total no of letter} = 7$$

$$\text{OEOEOEO} \quad \text{O} \rightarrow \text{Odd}$$

$$\text{E} \rightarrow \text{Even}$$

There are 3 even places and 4 Odd places so 3 vowels can fill in 3 even places and 4 consonant can fill in 4 Odd places. Then total number of ways

$$= {}^3 P_3 \cdot {}^4 P_4$$

$$= 3! \times 4!$$

$$= 6 \times 24$$

$$= 144$$

5. Given  ${}^n P_4 = 20 \cdot {}^n P_2$

$$\frac{|n}{|n-4} = 20 \cdot \frac{|n}{|n-2}$$

$$\text{Or } (n-2)(n-3) = 20$$

$$\text{Or } n^2 - 5n - 14 = 0$$

$$n^2 - 7n + 2n - 14 = 0$$

$$n(n-7) + 2(n-7) = 0$$

$$n = 7 \text{ or } n = -2 \text{ (not possible)}$$



6. Total school (n) = 6  
 No. of son (r) = 3  
 No. of ways =  ${}^n P_r = {}^6 P_3$

7. Words 'DRAUGHT'

If both vowels may not be separated this mean that both vowels comes together

D R G H T    A U  
 6 5 4 3 2        1

No. of ways if two vowels comes together  
 =  $6! \times 2! = 720 \times 2 = 1,440$

8. If  ${}^{13}C_6 + 2 \cdot {}^{13}C_5 + {}^{13}C_4 = {}^{15}C_x$   
 $({}^{13}C_6 + {}^{13}C_5) + ({}^{13}C_5 + {}^{13}C_4) = {}^{15}C_x$   
 ${}^{14}C_6 + {}^{14}C_5 = {}^{15}C_x$        $[{}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r]$   
 ${}^{15}C_6 = {}^{15}C_x$

On comparing,  $x = 6$

9. Let number of side of polygon is n

No. of diagonal =  $\frac{n(n-3)}{2}$

$\frac{44}{1} = \frac{n^2-3n}{2}$

$88 = n^2 - 3n$

$n^2 - 3n - 88 = 0$

$(n - 11) (n + 8) = 0$

If  $n - 11 = 0$  if  $n + 8 = 0$

$n = 11$        $n = -8$  (not valid)

10. In the word 'ARTICLE'

Total vowels = A, I, E (3)

Total consonant = R, T, C, L (4)

O E O E O E O

No. of ways =  ${}^3 P_3 \times 4!$

=  $3! \times 4!$

=  $6 \times 24$

= 144

## **DPP-NO-9A**

1. Given  $f(x) = {}^x C_3$

$$= \frac{|x|}{|3|x-3|}$$

$$F(x) = \frac{(x^3 - 3x^2 + 2x)}{6}$$

Differentiating w.r.t (x) both the sides, we get

$$f^1(x) = \frac{1}{6}(3x^2 - 6x + 2)$$

$$f^1(1) = \frac{1}{6}(3 \times 1^2 - 6 \times 1 + 2)$$

$$f^1(1) = \frac{-1}{6}$$

2. If  $y = x^x$

Taking log on both side

$$\text{Log } y = \log x^x$$

$$\text{Log } y = x \log x$$

Diff w. r. t (x)

$$\frac{1}{y} \frac{dy}{dx} = x \frac{1}{x} + \log x \cdot 1$$

$$\frac{1}{y} \frac{dy}{dx} = 1 + \log x$$

$$\frac{dy}{dx} = y(1 + \log x)$$

Diff again w. r. t 'x'

$$\frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} \{y(1 + \log x)\}$$

$$= y \cdot \frac{d}{dx}(1 + \log x) + (1 + \log x) \frac{dy}{dx}$$

$$= (1 + \log x) \frac{dy}{dx} + y \cdot \frac{d}{dx}(1 + \log x)$$

$$= \frac{dy}{dx}(1 + \log x) + y \cdot \frac{d}{dx}(1 + \log x)$$

3.  $y = e^{a \log x} + e^{x \log a}$

$$y = e^{\log x^a} + e^{\log a^x}$$

$$y = x^a + a^x$$

$$[e^{\log m} = m]$$

Diff w. r. t (x) both the sides we get

$$\frac{dy}{dx} = a x^{a-1} + a^x \log a$$

4. Given

$$y = x^3 - 3x$$

diff w. r. t 'x'

$$\frac{dy}{dx} = 3x^2 - 3 \quad \text{_____} (1)$$

$$0 = 3(x^2 - 1)$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

Diff (1) w. r. t 'x'

$$\frac{d^2y}{dx^2} = \frac{d}{dx} (3x^2 - 3)$$

$$\frac{d^2y}{dx^2} = 6x$$

$$\left(\frac{d^2y}{dx^2}\right)_{(x=\pm 1)} = 6(\pm 1) = \pm 6$$

5. Given curve

$$F(x) = x^3 - 2x + 3$$

$$\text{i.e } y = x^3 - 2x + 3$$

$$\frac{dy}{dx} = \frac{d}{dx} (x^3 - 2x + 3)$$

$$\frac{dy}{dx} = 3x^2 - 2$$

$$\left(\frac{dy}{dx}\right)_{(2,7)} = 3(2)^2 - 2$$

$$= 12 - 2$$

$$\left(\frac{dy}{dx}\right)_{(2,7)} = 10$$

$$\text{Slope of tangent } m = \left(\frac{dy}{dx}\right)_{(2,7)} = 10$$

The equation of tangent at (2, 7)

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 10(x - 2)$$

$$y - 7 = 10x - 20$$

$$y = 10x - 20 + 7$$

$$y = 10x - 13$$

6. Given,

$$y = \log\left(\frac{5-4x^2}{3+5x^2}\right)$$

$$y = \log(5-4x^2) - \log(3+5x^2)$$

diff w. r. t 'x'

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \log(5-4x^2) - \frac{d}{dx} \log(3+5x^2) \\ &= \frac{1}{(5-4x^2)} \frac{dy}{dx} (5-4x^2) - \frac{1}{(3+5x^2)} \frac{dy}{dx} (3+5x^2) \\ &= \frac{-8x}{5-4x^2} \frac{1}{(5-4x^2)} (0-8x) - \frac{1}{(3+5x^2)} (0+10x) \\ &= -\frac{10x}{(3+5x^2)} \\ &= \frac{-8x}{4x^2-5} - \frac{10x}{(3+5x^2)}\end{aligned}$$

7. If  $y = \log_y x$

$$y = \frac{\log x}{\log y}$$

$$y \log y = \log x$$

diff w. r. t x

$$\begin{aligned}y \frac{1}{y} \frac{dy}{dx} + \log y \cdot \frac{dy}{dx} &= \frac{1}{x} \\ \frac{dy}{dx} (1 + \log y) &= \frac{1}{x} \\ \frac{dy}{dx} &= \frac{1}{x(1+\log y)} = \frac{1}{(x+x \log y)}\end{aligned}$$

8. If  $x = \log t$ , diff w.r.t 't'

$$\frac{dx}{dt} = \frac{d}{dt} \log t = \frac{1}{t}$$

$$\text{And } y = e^t$$

$$\frac{dy}{dt} = \frac{d}{dt} (e^t) = e^t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^t}{1/t} = te^t$$

9. Given  $y = x^3 - x^2 x + 1$  .....(i)

$$\text{Diff w.r.f(x)} \frac{dy}{dx} = 3x^2 - 2x - 1$$

Since tangent is parallel to x - axis

$$\frac{dy}{dx} = 0$$

$$3x^2 - 2x - 1 = 0$$

$$3x^2 - 3x + x - 1 = 0$$

$$3x(x-1) + 1(x-1) = 0$$

$$(x - 1)(3x + 1) = 0$$

If  $x - 1 = 0$  and if  $3x + 1 = 0$

$$x = 1 \quad \text{and} \quad x = -1/3$$

$x = 1$  in equation (i) we get

$$\begin{aligned} y &= \left(\frac{-1}{3}\right)^3 - \left(\frac{-1}{3}\right)^2 - \left(\frac{-1}{3}\right) + 1 \\ &= \frac{-1}{27} - \frac{1}{9} + \frac{1}{3} + 1 \\ &= \frac{-1-3+9+27}{27} \end{aligned}$$

$$y = \frac{32}{27}$$

points are  $(-1/3, 32/27)$  and  $(1, 0)$

$$10. y = 1 + \frac{x}{|1|} + \frac{x^2}{|2|} + \dots + \frac{x^n}{|n|} + \dots \infty$$

$$\frac{dy}{dx} = 0 + \frac{1}{|1|} + \frac{2x}{|2|} + \dots + \frac{nx^{n-1}}{|n|} + \dots \infty$$

$$\frac{dy}{dx} = 1 + \frac{2x}{2|1|} + \dots + \frac{nx^{n-1}}{n|n-1|} + \dots \infty$$

$$\frac{dy}{dx} = 1 + \frac{x}{|1|} + \frac{x^2}{|2|} + \dots \infty$$

$$\frac{dy}{dx} = y$$

$$\frac{dy}{dx} - y = 0$$

## DPP-NO-9B

1. Given

$$x^p y^q = (x + y)^{p+q}$$

$$\log (x^p y^q) = \log (x + y)^{p+q}$$

$$\log x^p + \log y^q = (p + q) \log (x + y)$$

$$p \log x + q \log y = (p + q) \log (x + y)$$

diff w.r.t 'x'

$$p \times \frac{1}{x} + q \times \frac{1}{y} \frac{dy}{dx} = (p + q) \frac{1}{(x+y)} \frac{d}{dx} (x + y)$$

$$\frac{p}{x} + \frac{q}{y} \frac{dy}{dx} = \frac{(p+q)}{(x+y)} \left[ 1 + \frac{dy}{dx} \right]$$

$$\frac{p}{x} + \frac{q}{y} \frac{dy}{dx} = \frac{p+q}{x+y} \left[ \frac{p+q}{x+y} \right] \frac{dy}{dx}$$

$$\frac{q}{y} \frac{dy}{dx} - \left( \frac{p+q}{x+y} \right) \frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\frac{dy}{dx} \left[ \frac{q}{y} - \frac{(p+q)}{(x+y)} \right] = \frac{(p+q)x - p(x+y)}{(x+y)x}$$

$$\frac{dy}{dx} \left[ \frac{q(x+y) - y(p+q)}{y(x+y)} \right] = \frac{px + qx - px - py}{(x+y)x}$$

$$\frac{dy}{dx} \left[ \frac{qx + qy - py - qy}{y(x+y)} \right] = \frac{(qx - py)}{(x+y)x}$$

$$\frac{dy}{dx} \frac{(qx - py)}{y} = \frac{(qx - py)}{x}$$

$$\frac{dy}{dx} = \frac{y (qx - py)}{x (qx - py)}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

2.  $e^{xy} - 4xy = 4$

diff w.r.t 'x' on both side

$$\frac{d}{dx} (e^{xy} - 4xy) = \frac{d}{dx} (4)$$

$$\frac{d}{dx} e^{xy} - \frac{d}{dx} (4xy) = 0$$

$$e^{xy} \frac{d}{dx} (xy) - 4 \frac{d}{dx} (xy) = 0$$

$$\frac{d}{dx} (xy) [e^{xy} - 4] = 0$$

$$e^{xy} - 4 = 0$$

$$e^{xy} = 4$$

diff w.r.t 'x'

$$\frac{d}{dx} e^{xy} = \frac{d}{dx} (4)$$

$$e^{xy} \times \frac{d}{dx} (xy) = 0$$

$$e^{xy} \times \left[ x \times \frac{dy}{dx} + y \cdot 1 \right] = 0$$

$$x \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

3. if  $u = 3t^4 + 5t^3 + 2t^2 + t + 4$

diff w.r.t (t)

$$\begin{aligned} \frac{du}{dx} &= \frac{d}{dt} (3t^4 + 5t^3 + 2t^2 + t + 4) \\ &= \frac{d}{dt} (3t^4) + \frac{d}{dt} (5t^3) + \frac{d}{dt} (2t^2) + \frac{d}{dt} (t) + \frac{d}{dt} (4) \\ &= 3 \cdot 4t^3 + 5 \cdot 3t^2 + 2 \cdot 2t + 1 + 0 \\ &= 12t^3 + 15t^2 + 4t + 1 \end{aligned}$$

$$\begin{aligned} \left(\frac{du}{dx}\right)_{t=-1} &= 12(-1)^3 + 15(-1)^2 + 4(-1) + 1 \\ &= 12(-1) + 15(1) - 4 + 1 \\ &= -12 + 15 - 4 + 1 \\ &= -16 + 16 \\ &= 0 \end{aligned}$$

4.  $y = ae^{nx} + be^{-nx}$

diff w.r.t 'x'

$$\frac{dy}{dx} = ae^{nx} \cdot n + be^{-nx} \cdot (-n)$$

$$\frac{dy}{dx} = n ae^{nx} - n be^{-nx}$$

diff w.r.t 'x'

$$\begin{aligned} \frac{d^2y}{dx^2} &= n a e^{nx} \cdot n - n b e^{-nx} \cdot (-n) \\ &= n^2 (ae^{nx} + be^{-nx}) \end{aligned}$$

$$\frac{d^2y}{dx^2} = n^2 y$$

5. given curve

$$y = \frac{x-1}{x+2}$$

$$\frac{dy}{dx} = \frac{(x+2) \frac{d}{dx}(x-1) - (x-1) \frac{d}{dx}(x+2)}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{(x+2)(1-0) - (x-1)(1+0)}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{x+2-x+1}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{3}{(x+2)^2}$$

$$\left(\frac{dy}{dx}\right)_{(x=2)} = \frac{3}{(x+2)^2} = \frac{3}{(4)^2} = \frac{3}{16}$$

Slope of tangent =  $\left(\frac{3}{16}\right)$

6. if  $y = \sqrt{\frac{1-x}{1+x}}$

$$y = \frac{\sqrt{1-x}}{\sqrt{1+x}}$$

diff w.r.t 'x'

$$\frac{dy}{dx} = \frac{\sqrt{1+x} \frac{d}{dx} \sqrt{1-x} - \sqrt{1-x} \frac{d}{dx} \sqrt{1+x}}{(\sqrt{1+x})^2}$$

$$\frac{dy}{dx} = \frac{\sqrt{1+x} \cdot \frac{1}{2\sqrt{1-x}} \cdot (-1) - \sqrt{1-x} \cdot \frac{1}{2\sqrt{1+x}} \cdot (1)}{(1+x)}$$

$$\frac{dy}{dx} = \frac{(1+x)(-1) - (1-x) \cdot 1}{2\sqrt{1-x}\sqrt{1+x}(1+x)}$$

$$\frac{dy}{dx} = \frac{-1-x-x+1}{2\sqrt{1-x^2}(1+x)}$$

$$\frac{dy}{dx} = \frac{2}{2\sqrt{1-x^2}(1+x)}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}(1+x)} \cdot \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-1\sqrt{(1+x)(1-x)}}{(1-x^2)(1+x)}$$

$$\frac{dy}{dx} = \frac{-\sqrt{1+x}\sqrt{1-x}}{(1-x^2)\sqrt{1+x}\sqrt{1-x}}$$

$$\frac{dy}{dx} = -\sqrt{\frac{1-x}{1+x}} \cdot \frac{1}{(1-x^2)}$$

$$\frac{dy}{dx} = \frac{-y}{(1-x^2)}$$

$$\frac{dy}{dx} = \frac{y}{(x^2-1)}$$

7. given, slope =  $3x - 4$

$$\frac{dy}{dx} = 3x - 4$$

$$Dy = (3x - 4) dx$$

On integration

$$\int dy = \int (3x - 4) dx$$

$$y = \frac{3x^2}{2} - 4x + c \quad \text{_____ (1)}$$

it passes through (1, 2) then

$$2 = \frac{3(1)^2}{2} - 4 \times 1 + c$$

$$2 = \frac{3}{2} - 4 + c$$

$$c = 2 + 4 - \frac{3}{2}$$



$$c = \frac{9}{2} \text{ in equation (1)}$$

$$y = \frac{3x^2}{2} - 4x + \frac{9}{2}$$

$$2y = 3x^2 - 8x + 9$$

8. Given,  $x = at^3 + bt^2 - t$  and  $y = at^2 - 2bt$

$$\frac{dx}{dt} = a \cdot 3t^2 + b \cdot 2t - 1 \text{ and } \frac{dy}{dt} = 2at - 2b$$

$$= 2at^2 + 2bt - 1$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2at-2b}{3at^2+2bt-1}$$

$$\left(\frac{dy}{dx}\right)_{t=0} = \frac{2a \times 0 - 2b}{3a(0)^2 + 2b(0) - 1}$$

$$= \frac{0 - 2b}{0 + 0 - 1}$$

$$= \frac{-2b}{-1}$$

$$= 2b$$

9. If  $x^y = e^{x-y}$

Taking log on both side

$$\text{Log } x^y = \text{log } e^{x-y}$$

$$y \log x = (x - y) \log_e$$

$$y \log x = x - y \quad (\log_e = 1)$$

$$y \log x + y = x$$

$$y (\log x + 1) = x$$

$$y = \frac{x}{(\log x + 1)}$$

diff w.r.f (x)

$$\frac{dy}{dx} = \frac{(\log x + 1) \frac{d}{dx}(x) - x \cdot \frac{d}{dx}(\log x + 1)}{(\log x + 1)^2}$$

$$= \frac{(\log x + 1)1 - x \cdot \left(\frac{1}{x} + 0\right)}{(\log x + 1)^2}$$

$$= \frac{\log x + 1 - 1}{(\log x + 1)^2}$$

$$= \frac{\log x}{(\log x + 1)^2}$$

10. If  $y = 1 + \frac{x}{|1} + \frac{x^2}{|2} + \frac{x^3}{|3} + \dots \dots \dots \infty$

Diff w.r.t 'r'

$$\frac{dy}{dx} = \frac{d}{dx} \left[ 1 + \frac{x}{|1} + \frac{x^2}{|2} + \frac{x^3}{|3} + \dots \dots \dots \infty \right]$$

$$= \frac{d}{dx} (1) + \frac{d}{dx} \frac{x}{|1} + \frac{d}{dx} \frac{x^2}{|2} + \frac{d}{dx} \frac{x^3}{|3} + \dots \dots \dots \infty$$

$$= 0 + \frac{1}{|1} + \frac{2x}{|2} + \frac{3x^2}{|3} + \dots \dots \dots \infty$$

$$= 0 + 1 + \frac{2x}{2|1} \cancel{\frac{3x^2}{3|2}} \cancel{4} \dots \dots \dots \infty$$

$$= 1 + \frac{x}{|1} + \frac{x^2}{|2} + \dots \dots \dots \infty$$

$$\frac{dy}{dx} = y$$

## DPP-NO-9C

1. If  $f(x) = \log_e \left( \frac{x-1}{x+1} \right)$

Diff w.r.t 'x'

$$\frac{d}{dx} f'(x) = \frac{d}{dx} \log_e \left( \frac{x-1}{x+1} \right)$$

$$F'(x) = \frac{1}{(x-1)} - \frac{1}{(x+1)}$$

Given  $f'(x) = 1$

$$1 = \frac{x+1-x+1}{(x-1)(x+1)}$$

$$\frac{1}{1} = \frac{2}{x^2-1}$$

$$x^2 - 1 = 2$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

2. If  $x = at^2$  and  $y = 2at$  then  $\left( \frac{dy}{dx} \right)_{t=2} = ?$

Now given  $x = at^2$

Diff w.r.t 't'

$$\frac{dx}{dt} = \frac{d}{dt} (at^2) = a \frac{d}{dt} (t^2) = a \cdot 2t = 2at$$

and  $y = 2at$

Diff w.r.t 't'

$$\frac{dy}{dt} = \frac{d}{dt} (2at) = 2a \frac{d}{dt} (t) = 2a \cdot 1 = 2a$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

$$\left( \frac{dy}{dx} \right)_{t=2} = \frac{1}{2}$$

3. Given  $y = \log x^x$

$y = x \log x$

Diff w.r.t (x)

$$\frac{dy}{dx} = \frac{d}{dx} (x \log x)$$

$$= x \cdot \frac{d}{dx} (x \log x) + \log x \cdot \frac{d}{dx} (x)$$

$$= x \cdot \frac{1}{x} + \log x \cdot 1$$

$$= 1 + \log x$$

$$= \log^e + \log x$$

$$= \log ex$$

4.  $\frac{d}{dx} 2^{\log 2^x} = \frac{d}{dx} [x] [e^{\log_e x} = x]$

5. Given  $x = ct$  and  $y = \frac{c}{t}$

$$\begin{aligned} \frac{dx}{dt} &= \frac{d}{dt}(t) & \frac{dy}{dt} &= \frac{d}{dt} ct^{-1} \\ &= c \frac{d}{dt}(t) & &= c \cdot (-1) t^{-2} \\ &= c & &= \frac{-c}{t^2} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy/dt}{dx/dt} & &= \frac{-c/t^2}{c} \\ &= \frac{-c}{t^2 \cdot c} \\ &= -\frac{1}{t^2} \end{aligned}$$

**DPP-NO-10A**

$$\begin{aligned}
 1. \int 2^{3x} \cdot 3^{2x} \cdot 5^x dx & \\
 &= \int 8^x \cdot 9^x \cdot 5^x dx \\
 &= \int (8 \cdot 9 \cdot 5)^x dx \\
 &= \int (360)^x dx \\
 &= \frac{(360)^x}{\log 360} + c \\
 &= \frac{2^{3x} \cdot 3^{2x} \cdot 5^x}{\log 360} + c
 \end{aligned}$$

$$2. \int a^{2x} dx = \frac{a^{2x}}{2 \log a} \qquad \left[ \int a^{kx} dx = \frac{a^{kx}}{k \log a} \right]$$

$$\begin{aligned}
 3. I &= \int_0^5 \frac{x^2}{x^2 + (5-x)^2} dx \qquad \dots\dots\dots(1) \\
 I &= \int_0^5 \frac{(5+0-x)^2}{(5+0-x)^2 + (5-5-0-x)^2} dx \qquad \left[ \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right] \\
 I &= \int_0^5 \frac{(5-x)^2}{(5-x)^2 + x^2} dx \qquad \dots\dots\dots(2)
 \end{aligned}$$

adding (1) & (2)

$$\begin{aligned}
 2I &= \int_0^5 \frac{x^2 + (5-x)^2}{x^2 + (5-x)^2} dx \\
 &= \int_0^5 dx \\
 &= [x]_0^5 \\
 2I &= (5 - 0) \\
 I &= \frac{5}{2}
 \end{aligned}$$

$$\begin{aligned}
 4. \int_0^2 |1-x| dx &= \int_0^1 |1-x| dx + \int_1^2 |1-x| dx \\
 &= \int_0^1 (1-x) dx + \int_1^2 -(1-x) dx \\
 &= + \int_0^1 (1-x) dx - \int_1^2 (1-x) dx \\
 &= + \left[ x - \frac{x^2}{2} \right]_0^1 - \left[ x - \frac{x^2}{2} \right]_1^2 \\
 &= + \left[ \left\{ 1 - \frac{(1)^2}{2} \right\} - \left\{ 0 - \frac{0^2}{2} \right\} \right] - \left[ \left\{ 2 - \frac{2^2}{2} \right\} - \left\{ 1 - \frac{1^2}{2} \right\} \right] \\
 &= + \left[ \left( 1 - \frac{1}{2} \right) - 0 \right] - \left[ (2 - 2) - \left( 1 - \frac{1}{2} \right) \right] \\
 &= + \left[ \frac{1}{2} \right] - \left[ 0 - 1 + \frac{1}{2} \right] \\
 &= + \frac{1}{2} + 1 - \frac{1}{2} \quad \text{✓} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
5. \int_0^{1/2} \frac{dx}{\sqrt{3-2x}} \\
&= \int_0^{1/2} (3-2x)^{-1/2} dx \\
&= \left[ \frac{(3-2x)^{-\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)} \left(\frac{1}{-2}\right) \right]_0^{1/2} \\
&= \left[ \frac{(3-2x)^{1/2}}{\frac{1}{2}} \left(\frac{1}{-2}\right) \right]_0^{1/2} \\
&= \left[ \sqrt{3-2x} \right]_0^{1/2} \\
&= - \left[ \sqrt{3-2 \times \frac{1}{2}} - \sqrt{3-2 \times 0} \right] \\
&= - \left[ \sqrt{2} - \sqrt{3} \right] \\
&= (\sqrt{3} - \sqrt{2})
\end{aligned}$$

$$\begin{aligned}
6. \int_0^2 x e^{x^2} dx \\
&= \int_0^2 e^{x^2} \times x dx \\
&\text{Let } x^2 = t \\
&2x dx = dt \\
&x dx = \frac{dt}{2} \\
&= \int_0^4 e^t \times \frac{dt}{2} \quad (t = x^2 = 2^2 = 4) \\
&= \frac{1}{2} \int_0^4 e^t dt \\
&= \frac{1}{2} [e^t]_0^4 = \frac{1}{2} [e^4 - e^0] = \frac{1}{2} [e^4 - 1]
\end{aligned}$$

$$\begin{aligned}
7. \int_1^2 \left( \frac{1-x}{1+x} \right) dx \\
&= - \int_1^2 \left( \frac{x-1}{x+1} \right) dx \\
&= - \int_1^2 \left( \frac{x+1-2}{x+1} \right) dx \\
&= - \int_1^2 \left[ \frac{x-1}{x+1} - \frac{2}{x+1} \right] dx \\
&= - \int_1^2 1 dx + \int_1^2 \frac{2}{x+1} dx \\
&= - [x]_1^2 + [2 \log(x+1)]_1^2 \\
&= - (2-1) + 2 [\log(2+1) - \log(1+1)] \\
&= -1 + 2 (\log 3 - \log 2) \\
&= -1 + 2 \log \left( \frac{3}{2} \right)
\end{aligned}$$

$$= 2 \log\left(\frac{3}{2}\right) - 1$$

8.  $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}} dx$

let  $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}} dx = dt$$

$$\frac{1}{\sqrt{x}} dx = 2dt$$

$$= \int_0^{\sqrt{2}} 3^t \cdot 2 dt$$

$$= 2 \int_0^{\sqrt{2}} 3^t dt$$

$$= 2 \left[ \frac{3^t}{\log_3 e} \right]_0^{\sqrt{2}}$$

$$= \frac{2}{\log_3 e} [3^t]_0^{\sqrt{2}} = \frac{2}{\log_3 e} [3^{\sqrt{2}} - 3^0]$$

$$= \frac{2}{\log_3 e} [3^{\sqrt{2}} - 1]$$

9.  $\int \frac{x}{(x^2+1)(x^2+2)} dx$

Let  $x^2 = t$

$$2x dx = dt$$

$$x dx = \frac{dt}{2}$$

$$= \int \frac{1}{(t+1)(t+2)} \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{1}{(t+1)(t+2)} dt$$

$$= \frac{1}{2} \int \left[ \frac{1}{(t+1)} - \frac{1}{(t+2)} \right] dt$$

$$= \frac{1}{2} \left[ \int \frac{1}{(t+1)} dt - \int \frac{1}{(t+2)} dt \right]$$

$$= \frac{1}{2} [\log(t+1) - \log(t+2)] + c$$

$$= \frac{1}{2} \left[ \log \frac{(t+1)}{(t+2)} \right] + c$$

$$= \frac{1}{2} \left[ \log \left( \frac{x^2+1}{x^2+2} \right) \right] + c$$

10.  $\int_1^2 \frac{x}{x^2+1} dx = \frac{1}{2} \int_1^2 \frac{2x}{x^2+1} dx$

$$= \frac{1}{2} [\log(x^2 + 1)]_1^2$$

$$= \frac{1}{2} [\log(2^2 + 1) - \log(1^2 + 1)]$$

$$= \frac{1}{2} [\log 5 - \log 2]$$

$$= \frac{1}{2} \log\left(\frac{5}{2}\right)$$

## DPP-NO-10B

$$\begin{aligned} 1. \quad & \int e^x [f(x) + f^1(x)] dx \\ &= \int e^x f(x) dx + \int e^x f^1(x) dx \\ &= \int f(x) \cdot e^x dx + \int e^x f^1(x) dx \\ &= f(x) \int e^x dx - \int \left( \frac{d}{dx} f(x) \int e^x dx \right) dx + \int e^x \cdot f^1(x) dx \\ &= f(x) \cdot e^x - \int f^1(x) \cdot e^x dx + \int e^x \cdot f^1(x) dx \\ &= e^x \cdot f(x) + c \end{aligned}$$

$$\begin{aligned} 2. \quad & I = \int x e^{x^2} dx \\ & \text{let } x^2 = t \\ & 2x dx = dt \\ & x dx = \frac{dt}{2} \\ & I = \int e^t \frac{dt}{2} \\ & I = \frac{1}{2} \int e^t dt \\ & = \frac{1}{2} e^t + c \\ & = \frac{1}{2} e^{x^2} + c \end{aligned}$$

$$\begin{aligned} 3. \quad & \int_1^2 \left( \frac{1-x}{1+x} \right) dx \\ &= \int_1^2 \left( \frac{1}{1+x} - \frac{x}{1+x} \right) dx \\ &= \int_1^2 \frac{1}{1+x} dx - \int_1^2 \frac{x}{1+x} dx \\ &= \int_1^2 \frac{1}{1+x} dx - \int_1^2 \left( \frac{1+x-1}{1+x} \right) dx \\ &= \int_1^2 \frac{1}{(1+x)} dx - \int_1^2 \left( 1 - \frac{1}{1+x} \right) dx \\ &= \int_1^2 \frac{1}{1+x} dx - \int_1^2 1 \times dx + \int_1^2 \frac{1}{1+x} dx \\ &= 2 \int_1^2 \frac{1}{1+x} dx - \int_1^2 1 dx \\ &= 2 [\log(1+x)]_1^2 - [x]_1^2 \\ &= 2 [\log(2+1) - \log(1+1)] - [2-1] \\ &= 2 [\log 3 - \log 2] - 1 \\ &= 2 \log \frac{3}{2} - 1 \end{aligned}$$



$$4. \int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}} dx$$

$$\text{let } \sqrt{x} = t$$

$$= \int_1^2 3^{\sqrt{x}} \frac{1}{\sqrt{x}} dx \frac{1}{2\sqrt{x}} dx = dt$$

$$\frac{1}{\sqrt{x}} dx = 2 dt$$

|   |   |            |
|---|---|------------|
| x | 0 | 2          |
| t | 2 | $\sqrt{2}$ |

$$= \int_0^{\sqrt{2}} 3^t \cdot 2 dt$$

$$= 2 \int_0^{\sqrt{2}} 3^t dt$$

$$= 2 \left[ \frac{3^t}{\log 3} \right]_0^{\sqrt{2}}$$

$$= 2 \left[ \frac{3^{\sqrt{2}}}{\log 3} - \frac{3^0}{\log 3} \right]$$

$$= 2 \left[ \frac{3^{\sqrt{2}} - 3^0}{\log 3} \right]$$

$$= \frac{2(3^{\sqrt{2}} - 1)}{\log_e 3}$$

$$5. I = \int_0^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2-x}} dx \dots\dots\dots(1)$$

$$I = \int_0^2 \frac{\sqrt{0+2-x}}{\sqrt{0+2-x} + \sqrt{2-(0+2-x)}} dx \quad \left[ \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

$$I = \int_0^2 \frac{\sqrt{2-x}}{\sqrt{2-x} + \sqrt{x}} dx \dots\dots\dots(2)$$

addly (1) and (2) we get

$$2I = \int_0^2 \left( \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2-x}} + \frac{\sqrt{2-x}}{\sqrt{2-x} + \sqrt{x}} \right) dx$$

$$2I = \int_0^2 \frac{(\sqrt{x} + \sqrt{2-x})}{(\sqrt{x} + \sqrt{2-x})} dx$$

$$2I = \int_0^2 1 dx$$

$$2I = [x]_0^2$$

$$2I = [2 - 0]$$

$$2I = 2$$

$$I = 1$$

6.  $\int_{-1}^1 (e^x - e^{-x}) dx$

Because, given function  $(e^x - e^{-x})$  is a ODD function

Say,  $f(x) = e^x - e^{-x}$

Then  $f(-x) = -(e^x - e^{-x}) = -f(x)$

So, the given function is "ODD"

7. Given :  $f(x) = 3x^2 - \frac{2}{x^3}$  dx

We know,  $f(x) = \int f(x) dx = \int (3x^2 - \frac{2}{x^3}) dx$

$F(x) = x^3 + x^{-2} + c$  .....(i)

Given  $f(1) = 0$

$F(1) = (1)^3 + (1)^{-2} + c$

$0 = 1 + 1 + c$

$C = -2$

By (1)

$F(x) = x^3 + x^{-2} - 2$

8.  $\int_{-1}^1 \frac{|x|}{x} dx$

$f(x) = \frac{|x|}{x}$

$f(-x) = \frac{|-x|}{-x} = \frac{|x|}{x} = \frac{-|x|}{x}$

$f(-x) = -f(x)$

so it is odd function

$\int_{-1}^1 \frac{|x|}{x} dx = 0$